ITMM conference

Information Technologies and Mathematical Modelling named after A.F. Terpugov

2022

October 25-29
Karshi, Uzbekistan
online/offline

Resource Loss Systems and Performance Analysis of Wireless Networks

Konstantin Samouylov

Director of Applied Mathematics and Communication Technologies Institute, RUDN University (Peoples' Friendships University of Russia)

Foreword to the book

” In June 1955 the International Teletraffic Congress (ITC) has been established by the global community of teletraffic scientists and engineers in Copenhagen. After many years of absence, a small delegation of Russian scientists was able to join ITC13, held in Copenhagen in June 1991, once again.”

”Indeed, not only Sir Isaac Newton, but all scientists are standing on the shoulders of giants and they are invited to share their knowledge, ideas, papers, and books. In that summer 1991 the participants of ITC13 have realized once again that their teletraffic research is substantially formed by famous scientists, such as Agner Krarup Erlang, Soren Asmussen, Gely P. Basharin, Pavel P. Bocharov, Jacob W. Cohen, Erol Gelenbe, Boris V. Gnedenko, Arne Jensen, Frank P. Kelly, David G. Kendall, Aleksandr J. Khinchin, Gennadi P. Klimov, Andrey N. Kolmogorov, Igor N. Kovalenko, Guy Latouche, Andrey A. Markov, Debasis Mitra, Marcel F. Neuts, Vaidyanathan Ramaswami, Anatoliy V. Skorohod, Ryszard Syski, Lajos Takàcs, Whard Whitt, and Peter Whittle, to name a few of them.”

Udo R. Krieger

PREFACE TO THE BOOK

"The global telecommunication network is the largest technological object ever created by human society. It is being constantly enhanced and improved to keep up with our needs to exchange, store and process huge, ever-growing amounts of data. Network performance directly affects the Quality of Service (QoS) of traffic flows and the Quality of Experience (QoE) perceived by users of communication services."

"In queueing theory, models known as multi-resource service systems with random resource requirements, or **ReLS (**short for **resource loss systems)**, have proved highly relevant, as they reflect particularly well the processes of allocating and sharing radio resources in LTE networks and 5G NR systems operating in millimeter-wave bands. Motivated by the potential of this framework, the scholars specializing in mathematical teletraffic theory have laid the foundations of the theory of ReLSes, which is the central theme of this book."

"We are grateful to our colleagues, full and associate professors of RUDN University and Moscow State University, who have been teaching side by side with us courses on probability theory, stochastic processes, queueing theory, mathematical teletraffic theory, multiservice network theory, multiservice network performance analysis, mobile network performance analysis, 5G network modeling and performance analysis."

Valeriy Naumov, Yuliya Gaidamaka, Natalia Yarkina, and Konstantin Samouylov

Content of the book

Part I. Preliminaries

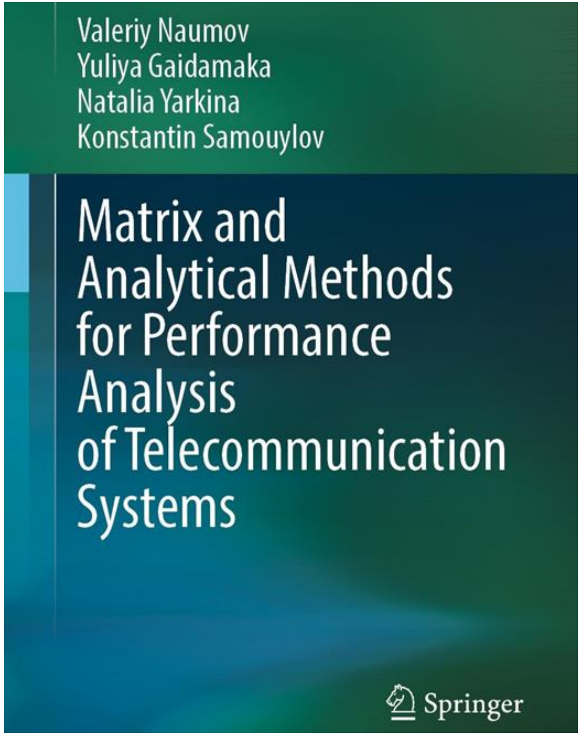
- Chapter 1. Modeling and performance analysis of telecommunication systems
- Chapter 2. Erlang’s systems
- Chapter 3. Multiservice loss networks
- Chapter 4. Modeling arrival and service processes

Part II. Matrix-analytical methods

- Chapter 5. Generator matrices
- Chapter 6. Block generator matrices
- Chapter 7. Matrix-geometric solutions

Part III. Resource queueing systems

- Chapter 8. Stochastic Lists of Resource Allocations
- Chapter 9. Multi-Resource Service System with Service Times Dependent on the Required Amounts of Resources
- Chapter 10. Markovian systems with random resource requirements
- Chapter 11. Stochastic networks with flexible servers
- Chapter 12. Application examples and case studies



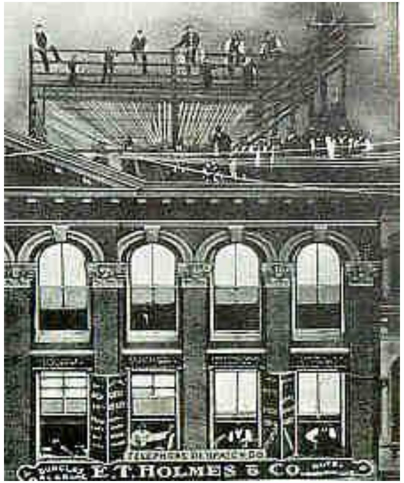
Content of the talk

- Loss Systems
- Loss Networks
- 5G/6G networks R&D process
- Loss Systems with Random Resource Requirements

Start of the Telecommunications Revolution

- The First Telephone and start of the Telecommunications Revolution – **1876**

Alexander Graham Bell: "Mr. Watson come here – I want to see you"



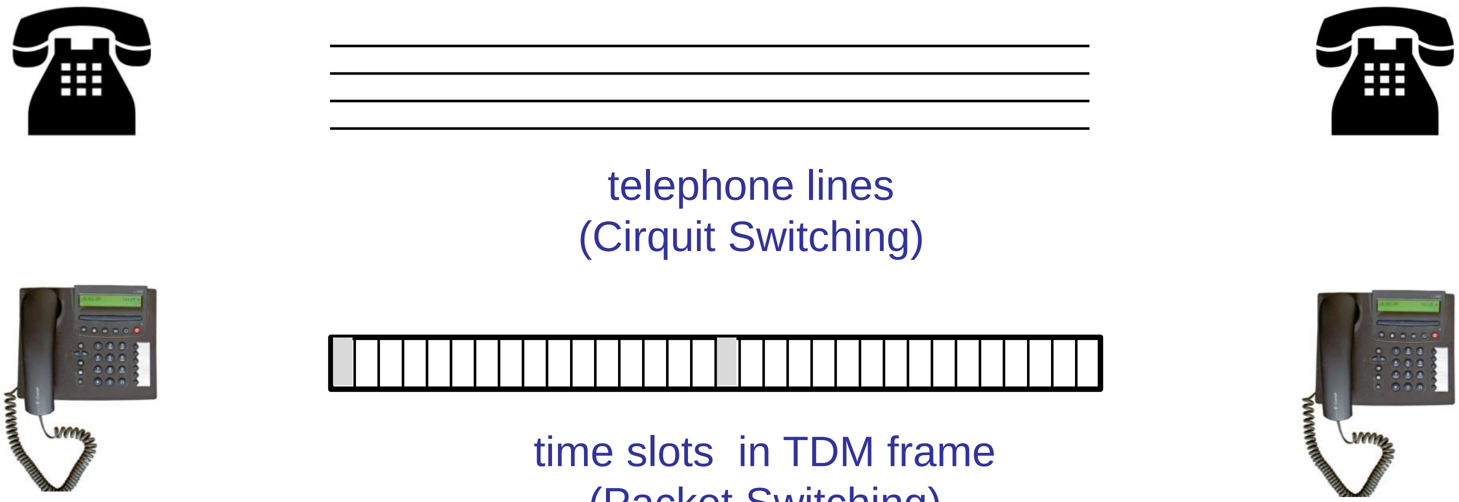
- The First Telephone Exchange – **1877**
- Agner Krarup Erlang was born in **1878**

Agner Krarup Erlang

- Agner Krarup Erlang (1878 – 1929) - the first person studying problems arising in the context of telephone calls.
  - ➢ The first paper on these problems – “*The theory of probability and telephone conversations*” (1909).
  - ➢ The most important work - "*Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges*" (1917).
- It is known that a researcher from the Bell Telephone Laboratories learned Danish in order to be able to read Erlang's papers in the original language.

TELEPHONE TRUNKS

- Trunking is a method for a system to provide network access to many customers by sharing a set of lines



- By studying a telephone trunk in 1917 Erlang worked out a formula, now known as *Erlang Loss Formula*.

Telephone Trunk Modelling

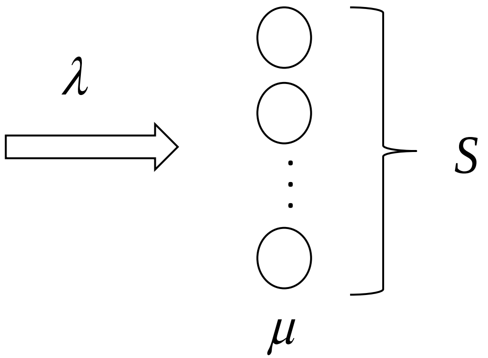
- S servers. Each of them is available if it is not busy;
- Arrival process is the Poisson with the rate λ, i.e. interarrival times α_i are independent and have exponential probability distribution with the mean $1/\lambda$,

$$P(\alpha_i > x) = \exp(-\lambda x), \quad x \geq 0, \quad i = 1,2,...$$

- Service times are independent and have exponential probability distribution with the mean $1/\mu$,

$$P(\beta_i > x) = \exp(-\mu x), \quad x \geq 0, \quad i = 1,2,...$$

- Arriving customer is lost if all servers are busy.



Equilibrium equations

- Transition rates up and down are the same

$$
\begin{cases}
\lambda p_0 = \mu p_1 \\
\quad \dots \\
\lambda p_{i-1} = i\mu p_i \\
\quad \dots \\
\lambda p_{S-1} = S\mu \; p_S \\
\sum_{i=0}^{S} p_i = 1
\end{cases}
$$

- The stationary probability distribution of the number of busy servers is given by

$$
p_n = \frac{\rho^n}{n!}\left(\sum_{k=0}^{S} \frac{\rho^k}{k!}\right)^{-1}, \; n = 0,1,\dots,S, \qquad (1)
$$

where $\rho = \lambda / \mu$ is the mean number of arrivals within the mean service time

Erlang Loss Formula

- Call blocking probability: probability that arriving call finds the system busy – propotion of the lost calls
- Time blocking probability: probability that at an arbitrary selected instant of time the system is busy – the proportion of time when the system is busy
- PASTA (Poisson Arrivals See Time Averages) property:

  If arrival process is Poisson then call and time blocking probabilities are the same.
- *Erlang's Loss Formula*: blocking probabilities are given by

$$\boxed{E_S(\rho) = \frac{\rho^S}{S!}\left(\sum_{k=0}^{S} \frac{\rho^k}{k!}\right)^{-1}} \qquad (2)$$

Computation of Loss Formula

- Computation: $E_S(\rho) = \dfrac{1}{C_S(\rho)}$ where

$$C_0(A) = 1\ , \quad C_i(\rho) = 1 + \left(\frac{i}{\rho}\right) C_{i-1}(\rho)\ , \ i = 1,2,\ldots,S$$

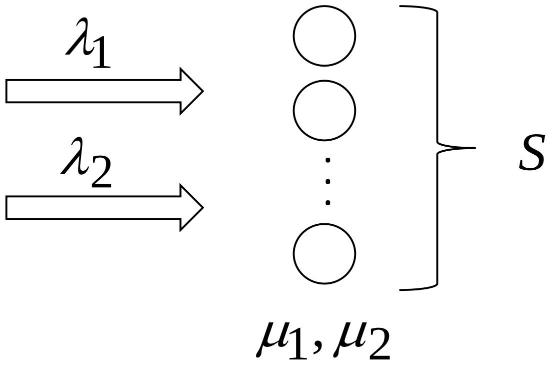
- Integral representation

$$E_S(\rho) = \frac{A^S e^{-\rho}}{\int_\rho^\infty e^{-t} t^S dt}$$

- *In 1957 Russian mathematician Boris Sevastyanov proved that Erlang Loss Formula remains valid if service times have general distribution.*

Two arriving processes

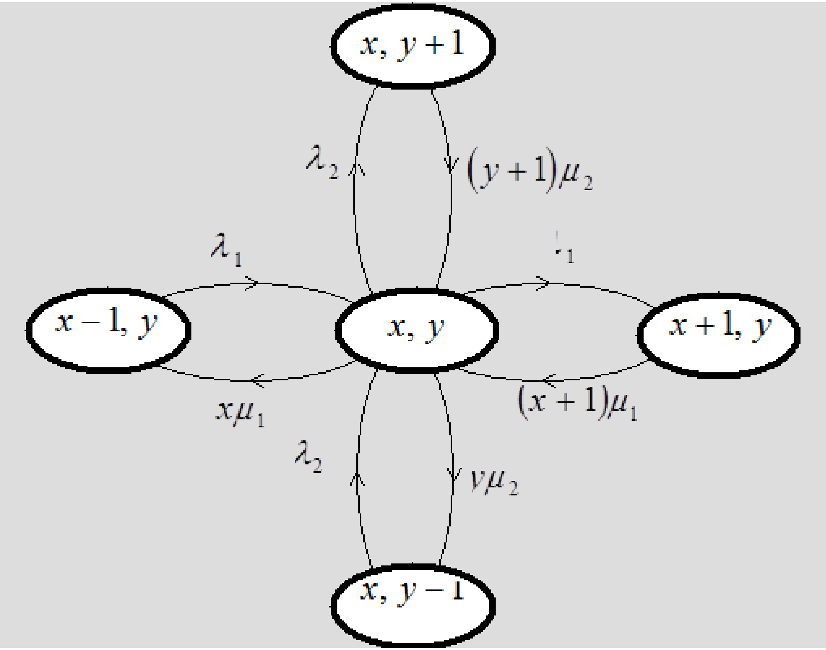
- S servers. Each of them is available if it is not busy;
- Two independent Poisson arrival processes with intensities λ_1 and λ_2
- Service times are independent and have exponential probability distributions with parameters μ_1 and μ_2
- Arriving customer is lost if all servers are busy.



State transition diagram

- The set of feasible states is

$$\mathcal{X} = \{(x, y) \in \mathbb{N}^2 \mid x, y \geq 0,\ x + y \leq S\}$$



RUDN university

Two-dimensional Erlang distribution

- The stationary probability distribution of the number of busy servers is given by

$$p_{x,y} = \frac{\frac{\rho_1^x}{x!}\frac{\rho_2^y}{y!}}{\sum_{i=0}^{S}\sum_{j=0}^{S-i}\frac{\rho_1^i}{i!}\frac{\rho_2^j}{j!}}, \quad x, y \geq 0, \; x+y \leq S. \qquad (3)$$

- Blocking probabilities are given by

$$\boxed{B_1 = B_2 = \frac{\sum_{i=0}^{S}\frac{\rho_1^i}{i!}\frac{\rho_2^{S-i}}{(S-i)!}}{\sum_{i=0}^{S}\sum_{j=0}^{S-i}\frac{\rho_1^i}{i!}\frac{\rho_2^j}{j!}} = E_S(\rho)} \qquad (4)$$

with $\rho = \rho_1 + \rho_2$

Multi-dimensional Erlang distribution

- Superposition of independent Poisson processes is the Poisson process with intensity

$$\lambda = \sum_{k=1}^{K} \lambda_k$$

- Probability distribution of the service time is the weighed sum of exponential distributions with the mean:

$$\frac{1}{\mu} = \sum_{k=1}^{K} \frac{\lambda_k}{\lambda}\left(\frac{1}{\mu_k}\right) = \frac{1}{\lambda}\sum_{i=1}^{K}\left(\frac{\lambda_k}{\mu_k}\right) = \frac{1}{\lambda}\sum_{k=1}^{K}\rho_k$$

- Since Erlang Loss Formula remains valid if service times have general distribution, blocking probability is given by $E_S(\rho)$ with

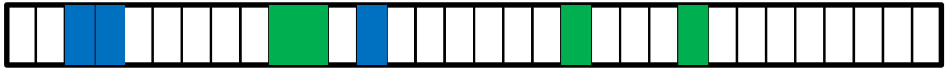
$$\rho = \frac{\lambda}{\mu} = \sum_{k=1}^{K}\rho_k$$

Generalized loss systems

- *Multi-class sources:* Class k customers arrive as a Poisson process with rate λ_k with the mean holding time $1/\mu_c$
- *Simultaneous acquisition of multiple servers:* A class k customer requires to hold s_k servers simultaneously.
- The set of feasible states is

$$\mathcal{X} = \{\mathbf{n} \in \mathbb{N}^K \mid \sum_{k=1}^{K} s_k n_k \le S\}$$

- Generalized loss systems are used for the performance analysis of high-speed data transmission, that requires multiple TDM slots



Generalized Loss Systems

- The stationary distribution is given by

$$\boxed{p_{\mathbf{n}} = \frac{1}{G}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!},\quad \mathbf{n}\in\mathcal{X}, \qquad G = \sum_{\mathbf{n}\in\mathcal{X}}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!},} \tag{5}$$

where

$$\rho_k = \lambda_k / \mu_k$$

- Blocking probabilities for class *i* customers can be calculated as

$$B_i = 1 - \frac{G_i}{G}, \tag{6}$$

where

$$G_i = \sum_{n_1 s_1 + \ldots + n_K s_K + s_i \le S}\ \prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!}$$

Loss networks

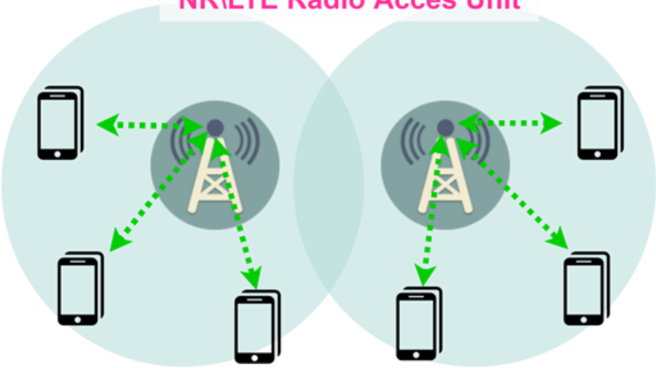
- Simultaneous acquisition of multiple servers of different types: There are S_m servers of type m. A class k customers requires to hold s_{km} serversof type m simultaneously.
- The preceding formulas for the stationary distribution are valid. Only the set of feasible states in different

$$\mathcal{X} = \{\mathbf{n} \in \mathbb{N}^K \mid \sum_{k=1}^{K} s_{km} n_k \le S_m, m = 1,2,\ldots M\}$$

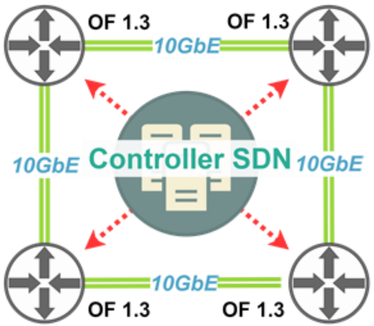
- The loss network provides a general model for a circuitswitched network that carries multi-rate traffic
- The model is equally applicable to bidirectional flows.
- The reverse traffic for a given pair of nodes may have different bandwidth requirements

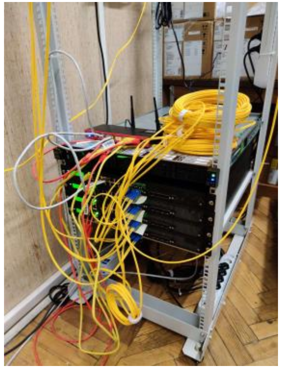
5G Lab RUDN: advanced R&D

NR\LTE Radio Acces Unit



Software Defined Network





Sensors & Actuators

Core Network

UAV Networks

5G/6G NETWORKS: R&D SINCE 2016 (1/3)

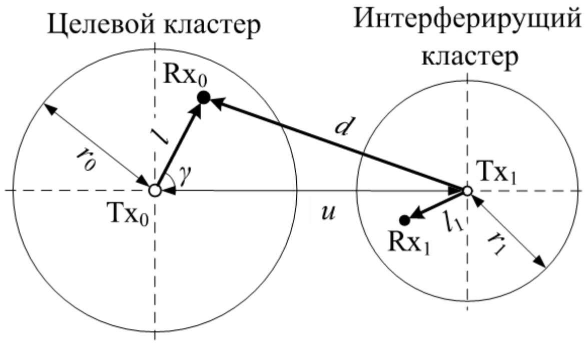
The book, which to a certain extent can be considered a textbook, was written based on the results of approximately five years of research in the field of 5G / 6G wireless networks, conducted by a team of scientists and experts - communications engineers from the Tampere University of Technology, Finland (now the University of Tampere, Tampere University, TAU) and applied mathematicians from the Peoples' Friendship University of Russia (RUDN). During this time, the members of the team defended four doctoral and more than ten dissertations for the degree of Candidate of Science and the degree of Doctor of Technology (PhD) on research topics. More than one hundred articles have been published (we included exactly one hundred of them in the list of references) in scientific journals of the first, highest quartile (Q1) according to the JCR Science Edition impact factor, more than twenty research projects in the field of 4G/5G/6G networks have been implemented. The book covers the lecture part of several special courses on wireless technologies and 5G networks given by the authors at the Peoples' Friendship University of Russia, the Higher School of Economics and TAU .

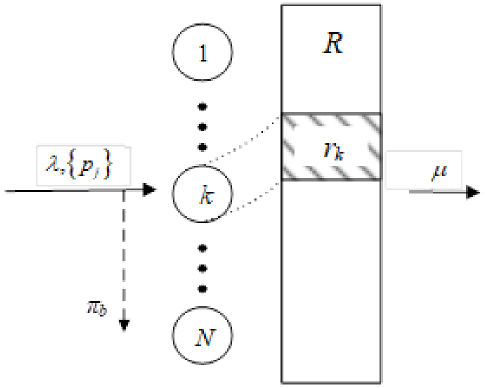
5G/6G NETWORKS: R&D SINCE 2016 (2/3)

The process of researching the performance of wireless networks is notoriously iterative. **First, it is proposed to investigate a specific, but quite general scenario of its application.** The scenario should include as much as possible the metrics of 5G networks, the study of which has not yet been carried out or almost not carried out, i.e. the tasks to be solved in research should be ultra-new. **Secondly, a system model is built that includes the components of 5G networks corresponding to the scenario,** such as a signal propagation model, a line-of-sight blocking model, an access model, a subscriber service model, etc. To analyze the performance metrics of a system model, three main modeling methods are used, depending on the need: analytical or statistical (Monte Carlo method) modeling; simulation modeling, and, may be, measurements on laboratory testbed. **Thirdly**, there comes the **stage of building models and developing methods for their analysis and calculation of the metrics of interest** in accordance with the studied scenario for the use of wireless technology and the developed system model. Due to the random nature of user request flows, random durations of user sessions, and random amounts of network resources occupied, in most cases, modeling is based on the apparatus of various sections of probability theory, primarily on queuing theory, the theory of Markov random processes, point random processes and methods of stochastic geometry. Studies have shown that in terms of the queuing theory, models called **resource queuing systems** turned out to be in demand. These systems make it possible to most adequately describe the processes of providing and sharing radio resources, for example, such as in 5G New Radio (5G NR) networks. Forecasts of the development of network technologies indicate that the models will be in demand in the future, and for this, specialists in the mathematical theory of teletraffic have already created the foundations of the theory of resource queuing systems. These models are supplemented by methods of point random processes and stochastic geometry, which allow taking into account the location of users and devices in space, random distances between them, the configuration of antennas and rooms, in the case of high-frequency technologies, the configuration of obstacles blocking the transmission (so-called blockers) and other parameters modeling. These parameters, as shown in this book, as well as in other sources, can be described geometrically, but taking into account the random arrangement of objects in terms of stochastic geometry. **Thus, a new discipline arises, which we called the theory of stochastic analysis of wireless networks**, and for which the book shows various aspects of its application to 5G / 6G networks.

5G/6G NETWORKS: R&D SINCE 2016 (3/3)

Part I explores models, characteristics, and methods of analysis of radio channels of 5G/6G networks. Here, in Chapter 2, models of the major components of these networks are shown, including radio propagation models, phased array models, line-of-sight path-blocking models, and link parameter models. Chapter 3 briefly outlines the **theoretical foundations for stochastic performance analysis of 5G/6G access networks**, including the main characteristics of radio access networks, the **stochastic geometry methods** used in the book, **queuing theory methods**, and direct device interaction models.





Loss systems with random resource requirements (1/3)

- Acquisition of multiple *resources of different types:*
  - There are R_m units of resources of type m, $\mathbf{R} = (R_1,\dots,R_M)$
  - The i-th customer of class k requires to hold $r_{mk}(i)$ units of resources of type m.
  - Resource demands $\mathbf{r}_k(i) = (r_{k1}(i),\dots,r_{kM}(i))$, $i = 1,2,\dots$ of class k customers are nonnegative random vectors with cumulative distribution functions $F_k(\mathbf{x})$
- The set of feasible states is given by

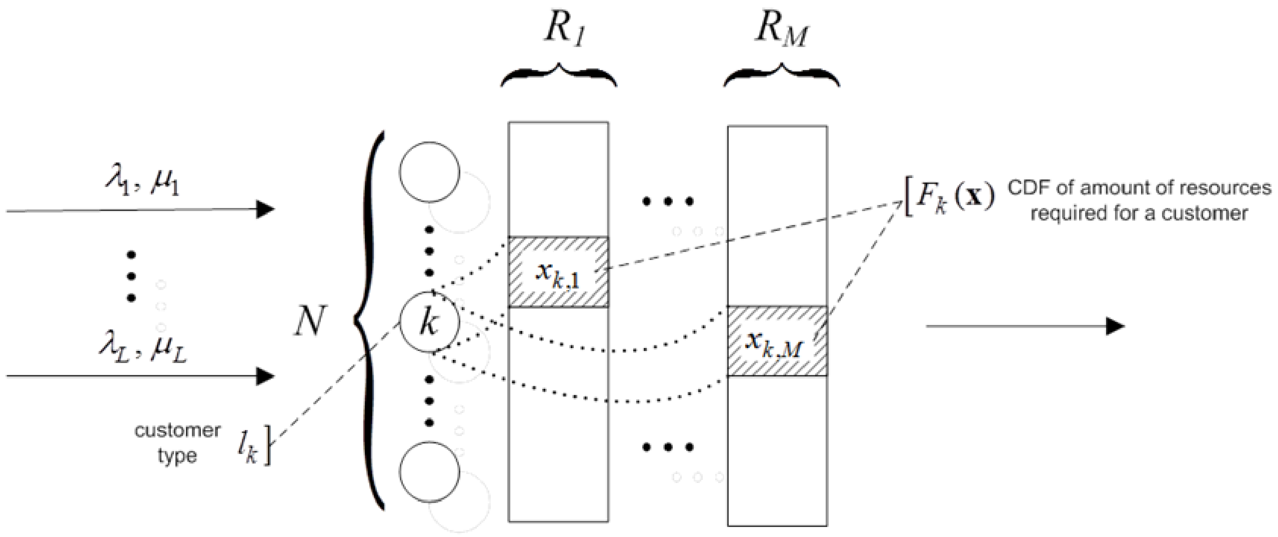
$$\mathcal{X} = \{(\mathbf{n},\boldsymbol{\gamma}_1,\dots,\boldsymbol{\gamma}_K) \mid \mathbf{n} \in \mathbb{N}^K, \boldsymbol{\gamma}_k \in \mathbb{R}_+^M, k = 1,2,\dots,K, \sum_{k=1}^{K} \boldsymbol{\gamma}_k \le \mathbf{R}, \sum_{k=1}^{K} n_k \le S\} \qquad (7)$$

$\mathbf{n} = (n_1,\dots,n_K)$ – population vector

$\boldsymbol{\gamma}_k = (\gamma_{k1},\dots,\gamma_{kM})$ – vector of resources occupied by class k customers

RUDN university

Loss systems with random resource requirements (2/3)



Loss systems with random resource requirements (3/3)

- Cumulative distribution functions of the stationary distribution are given by

$$\boxed{\begin{gathered} P_{\mathbf{n}}(\mathbf{x}_1,\ldots,\mathbf{x}_K)=\frac{1}{G}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!}F_k^{*n_k}(\mathbf{x}_k),\quad (\mathbf{n},\mathbf{x}_1,\ldots,\mathbf{x}_K)\in\mathcal{X}, \\ G=\sum_{n_1+\ldots+n_K\le S}(F_1^{*n_1}*\ldots*F_K^{*n_K})(\mathbf{R})\frac{\rho_1^{n_1}\cdots\rho_K^{n_K}}{n_1!\cdots n_K!} \end{gathered}} \qquad (8)$$

$*$ – convolution symbol

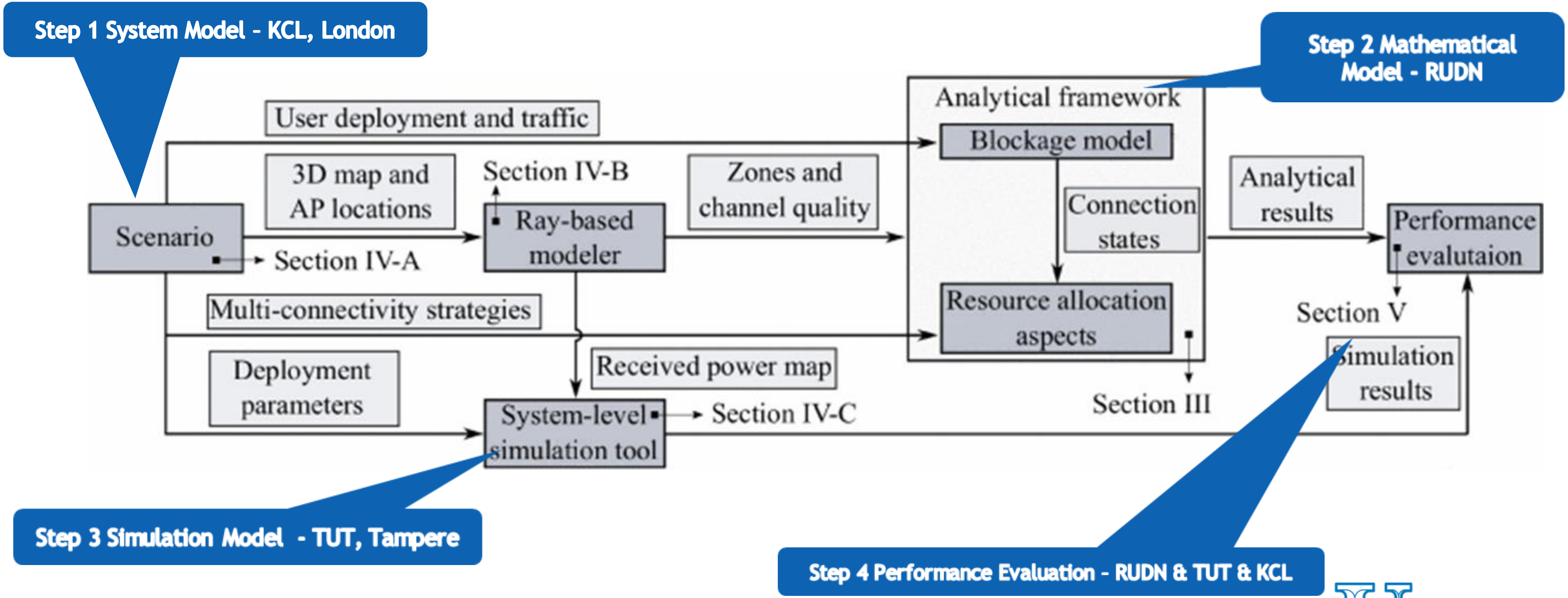
- Blocking probability of class k customers: $B_k=1-\dfrac{G_k}{G}$,

$$G_k=\sum_{n_1+\ldots+n_K<S}(F_1^{*n_1}*\ldots*F_k^{*(n_k+1)}*\ldots*F_K^{*n_K})(\mathbf{R})\frac{\rho_1^{n_1}\cdots\rho_K^{n_K}}{n_1!\cdots n_K!}$$

5G/6G NETWORKS: MODEL OF JOINT RESEARCH (1/4)

COLLABORATION – KING'S COLLEGE LONDON (KCL, UK), TAMPERE UNIVERSITY OF TECHNOLOGY (TUT, FINLAND), AND RUDN UNIVERSITY

PROBLEM - DYNAMIC MULTI-CONNECTIVITY PERFORMANCE IN ULTRA-DENSE URBAN MMWAVE DEPLOYMENTS

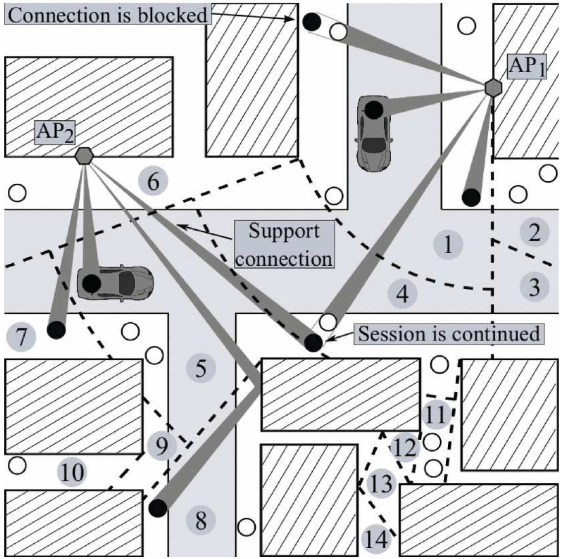


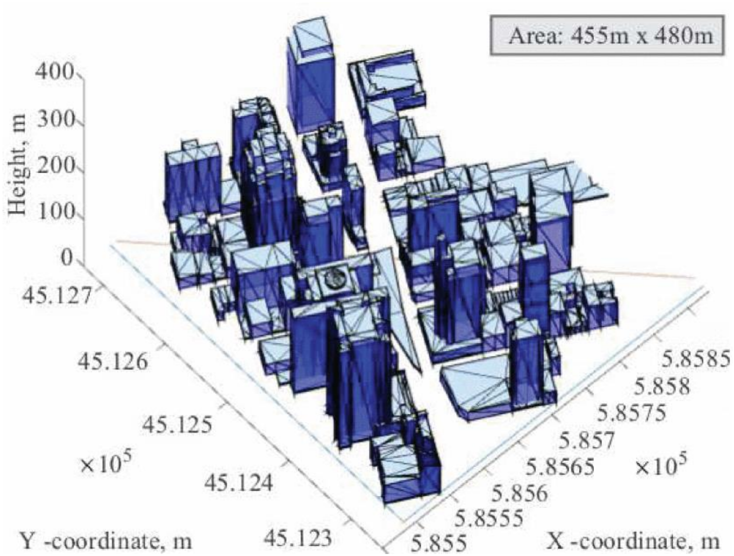
RUDN university

5G/6G NETWORKS: MODEL OF JOINT RESEARCH (2/4)

PROBLEM - DYNAMIC MULTI-CONNECTIVITY PERFORMANCE IN ULTRA-DENSE URBAN MMWAVE DEPLOYMENTS

Scenario of mmWave multi-connectivity in urban scenarios



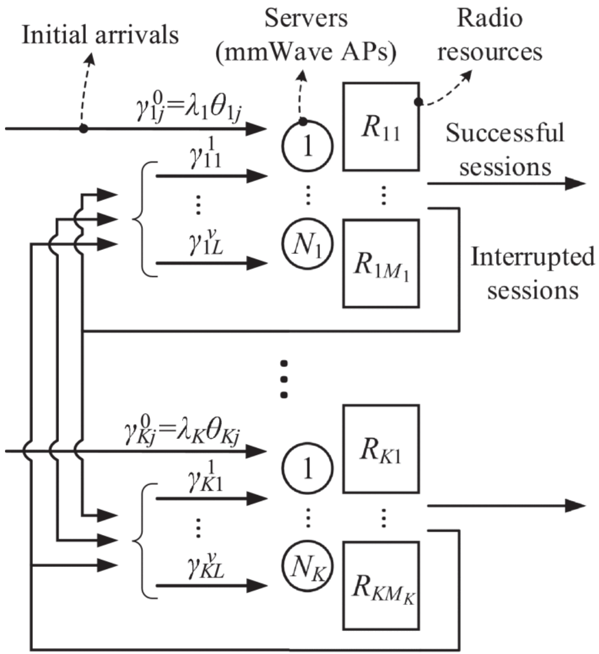


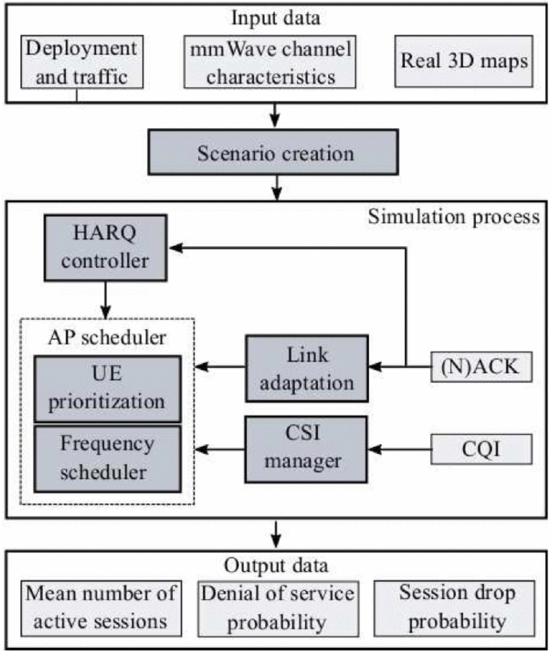
Detailed 3D model of the Times Square, NYC, USA

5G/6G NETWORKS: MODEL OF JOINT RESEARCH (3/4)

PROBLEM - DYNAMIC MULTI-CONNECTIVITY PERFORMANCE IN ULTRA-DENSE URBAN MMWAVE DEPLOYMENTS

An illustration of the proposed queuing network model

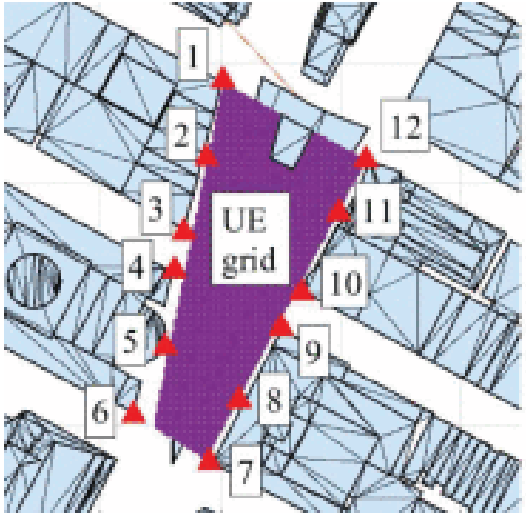




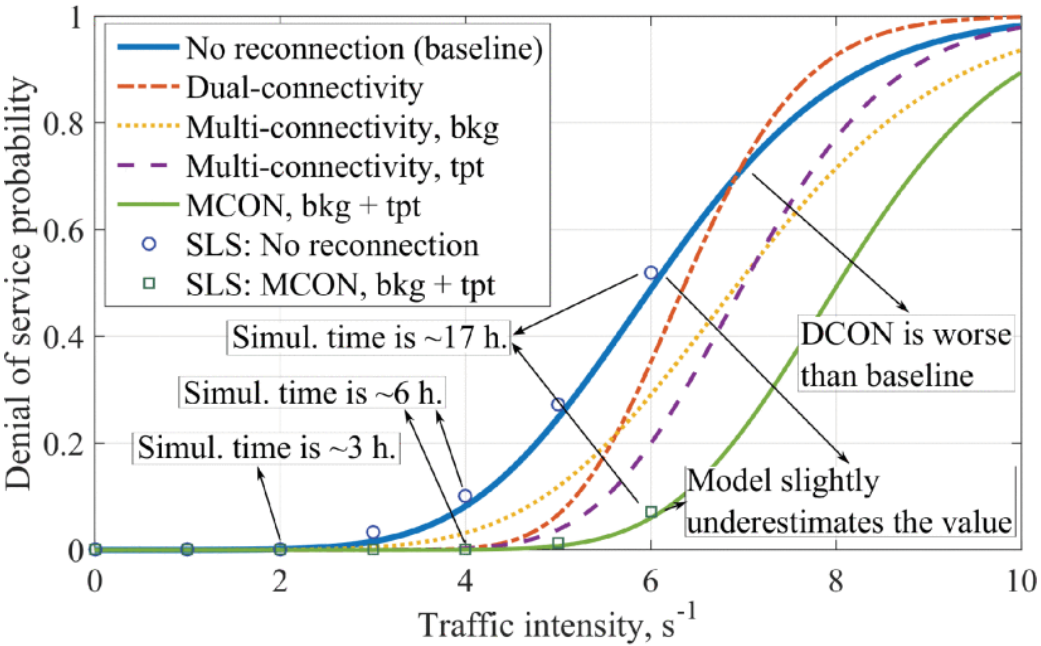
High-level structure of our system-level simulator

5G/6G NETWORKS: MODEL OF JOINT RESEARCH (4/4)

PROBLEM - DYNAMIC MULTI-CONNECTIVITY PERFORMANCE IN ULTRA-DENSE URBAN MMWAVE DEPLOYMENTS



Candidate deployment of access points

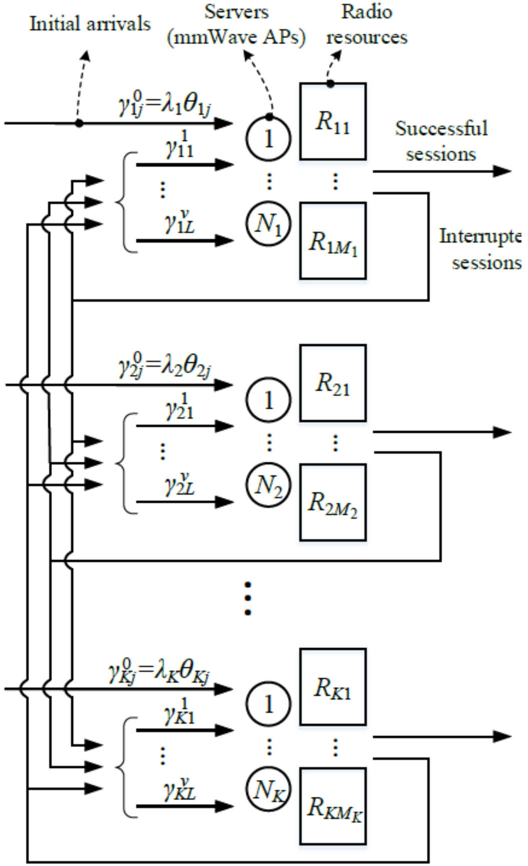


Denial of service probability vs. traffic intensity

RUDN university

Loss networks with random resource requirements and signals

- Network contains customers and signals
- Arriving signal interrupts the service of a customer and forces a customer to leave the network, or to move instantaneously to another loss system where the customer requests new service.
- If the service of a customer was not interrupted, the customer leaves the network and is considered as successfully served.



Some references

- *M. Stasiak, M. Glabowski, A. Wisniewski, P. Zwierzykowski*. Modelling and Dimensioning of Mobile Wireless Networks: From GSM to LTE. Wiley, 2010.
- *V. Naumov, K. Samuoylov*. On relationship between queuing systems with resources and Erlang networks, Informatics and Applications, 2016, Vol. 10, No. 3, 9-14
- *V. Naumov, K. Samouylov*. Analysis of multi-resource loss system with state dependent arrival and service rates. Probability in the Engineering and Informational Sciences. 2017, Vol. 31, No. 4, 413-419.
- *V. Naumov, Y. Gaidamaka, K. Samouylov*, On truncation of the matrix-geometric stationary distributions (2019) Mathematics, 7 (9), статья № 798. DOI: 10.3390/math7090798
- *V. Naumov, Y. Gaidamaka, K. Samouylov*, Computing the stationary distribution of queueing systems with random resource requirements via fast fourier transform (2020) Mathematics, 8 (5), статья № 772.
- *V. Naumov, K. Samouylov*. Resource system with losses in a random environment (2021) Mathematics, 9 (21), статья № 2685.

Contacts

Peoples' Friendship University of Russia
(RUDN University)

Miklukho-Maklaya st, 6,
Moscow, Russia 117198
tel. +7 495 952 2823

Konstantin Samouylov

Director of the Applied Mathematics & Communications Technology Institute

Head of the Applied Probability and Informatics Department

ksam@sci.pfu.edu.ru